

$$1) \quad \vec{\nabla} V = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z$$

$$\vec{\nabla} \times (\vec{\nabla} V) = \left(\frac{\partial^2 V}{\partial y \partial z} - \frac{\partial^2 V}{\partial z \partial y} \right) \hat{a}_x + \left(\frac{\partial^2 V}{\partial z \partial x} - \frac{\partial^2 V}{\partial x \partial z} \right) \hat{a}_y +$$

$$\left(\frac{\partial^2 V}{\partial x \partial y} - \frac{\partial^2 V}{\partial y \partial x} \right) \hat{a}_z = 0$$

$$\vec{\nabla} \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{a}_y +$$

$$\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{a}_z$$

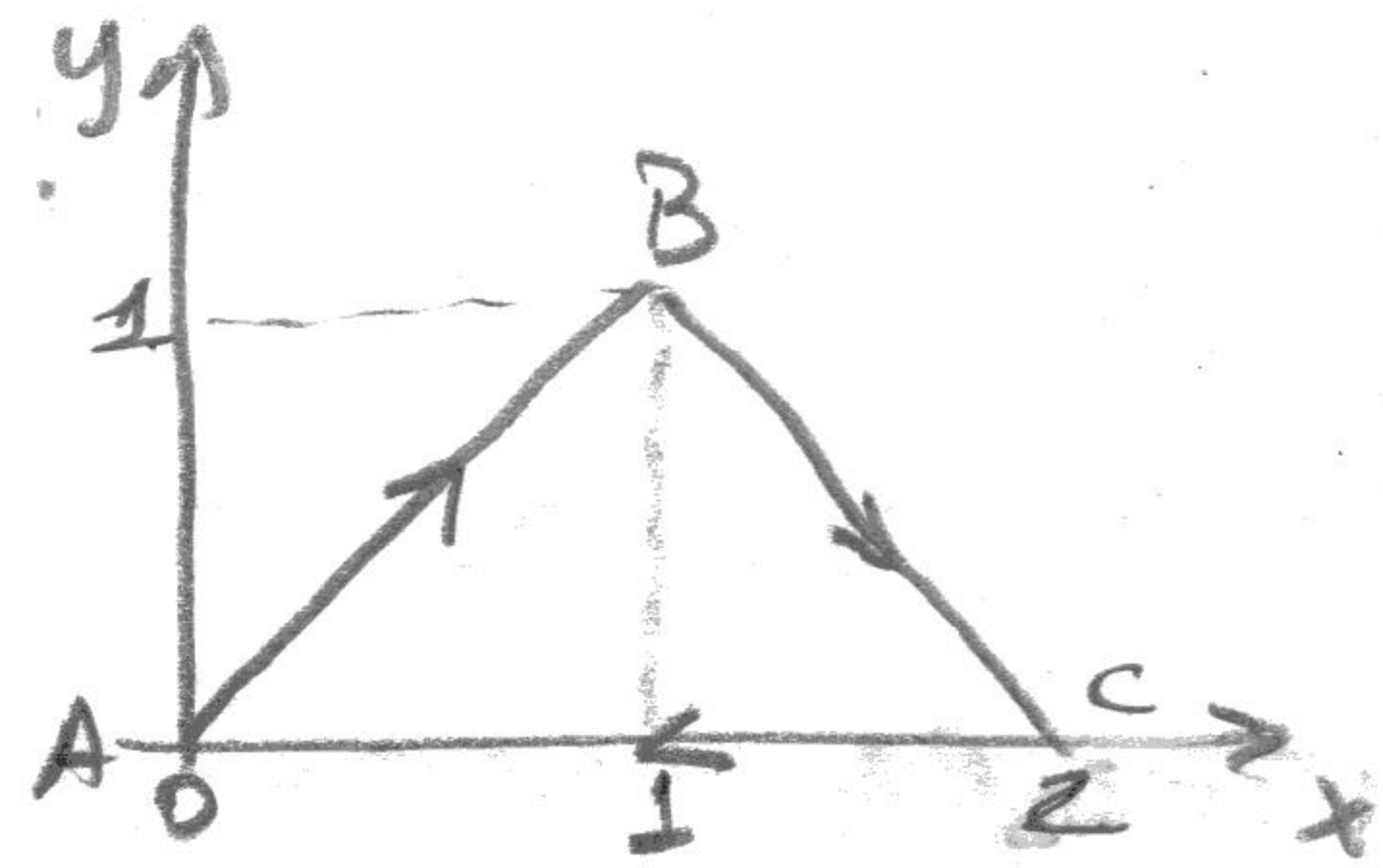
$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} + \frac{\partial^2 A_x}{\partial y \partial z} - \frac{\partial^2 A_z}{\partial y \partial x} +$$

$$\frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y} = 0$$

$$2) \vec{H} = x^2 y \hat{a}_x - y \hat{a}_y$$

$$a) A \rightarrow B \quad d\vec{\ell} = dx \hat{a}_x + dy \hat{a}_y \quad y = x$$

$$\int_A^B \vec{H} \cdot d\vec{\ell} = \int_A^B x^2 y dx - \int_A^B y dy = \int_0^1 x^2 x dx - \int_0^1 y dy = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$



$$B \rightarrow C \quad d\vec{\ell} = dx \hat{a}_x + dy \hat{a}_y \quad y = -x + 2 \quad \vec{H} \cdot d\vec{\ell} = x^2(-x+2) dx - y dy$$

$$\int_B^C \vec{H} \cdot d\vec{\ell} = \int_1^2 x^2(-x+2) dx - \int_1^0 y dy = \left(-\frac{x^4}{4} + 2\frac{x^3}{3} \right) \Big|_1^2 - \frac{y^2}{2} \Big|_1^0 = -\frac{2^4}{4} + \frac{1}{4} + \frac{2 \cdot 2^3}{3} - \frac{2}{3} + \frac{1}{2} = -\frac{13}{4} + \frac{14}{3}$$

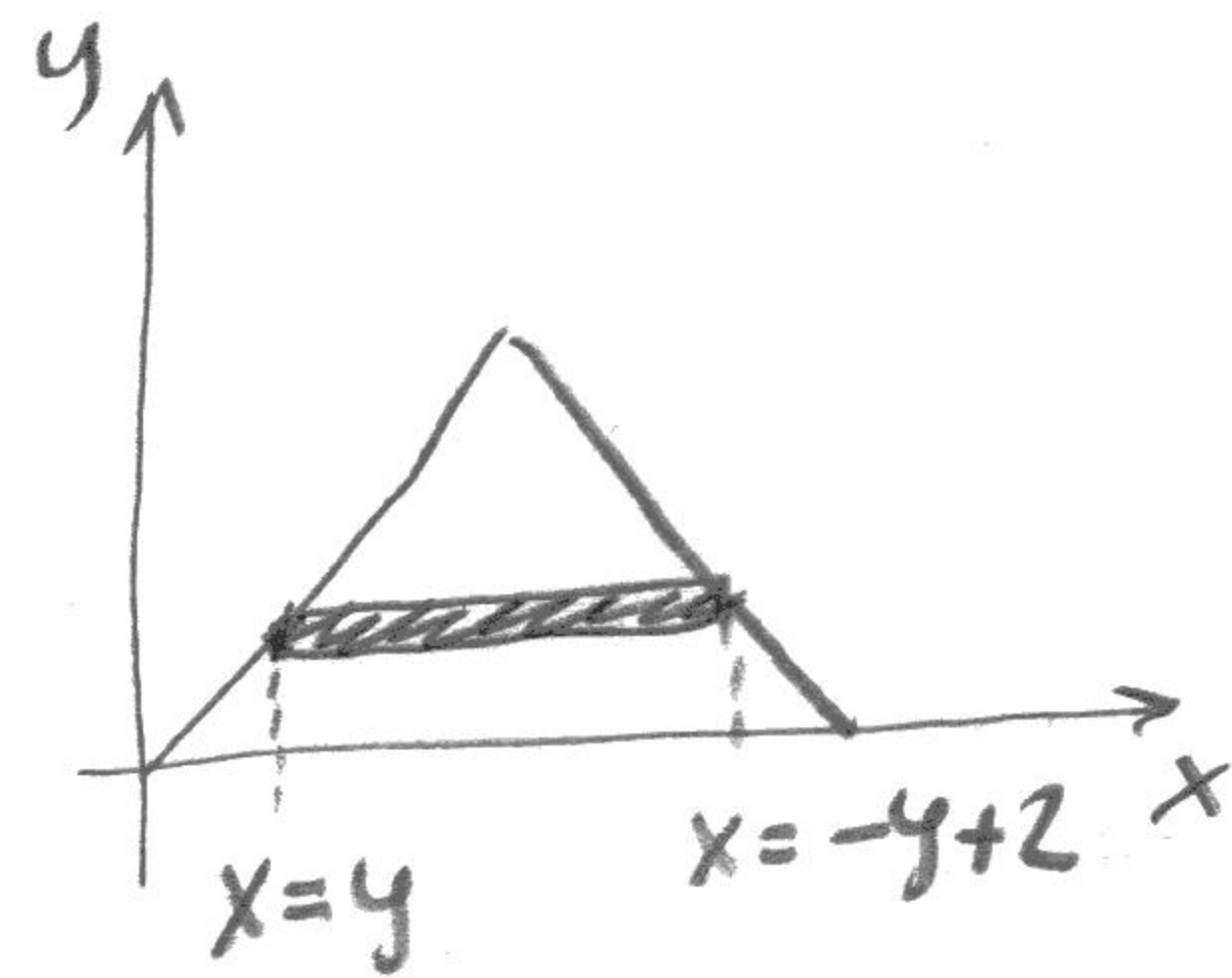
$$C \rightarrow A \quad d\vec{\ell} = dx \hat{a}_x + dy \hat{a}_y \quad y = 0 \Rightarrow \int_C^A \vec{H} \cdot d\vec{\ell} = 0$$

$$\oint \vec{H} \cdot d\vec{\ell} = -\frac{1}{4} - \frac{13}{4} + \frac{14}{3} = -\frac{7}{2} + \frac{14}{3} = \boxed{\frac{7}{6}}$$

$$b) \vec{\nabla} \times \vec{H} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{a}_z = (0 - x^2) \hat{a}_z = -x^2 \hat{a}_z$$

$$d\vec{S} = dx dy \hat{a}_x + dz dx \hat{a}_y + dx dy \hat{a}_z$$

$$(\vec{\nabla} \times \vec{H}) \cdot d\vec{S} = -x^2 dx dy$$



$$\int (\vec{\nabla} \times \vec{H}) \cdot d\vec{S} = \int_0^1 \int_y^{-y+2} (-x^2) dx dy =$$

$$= \int_0^1 \left(-\frac{x^3}{3} \right) \Big|_y^{-y+2} dy = \int_0^1 \left[-\frac{(-y+2)^3}{3} + \frac{y^3}{3} \right] dy = -\frac{1}{3} \left[-\frac{(-y+2)^4}{4} + \frac{y^4}{4} \right] \Big|_0^1 =$$

$$= -\frac{1}{3} \left[-\left(\frac{1}{4} - \frac{2^4}{4} \right) + \frac{1}{4} \right] = -\frac{1}{3} \cdot \frac{7}{2} = \boxed{-\frac{7}{6}}$$

c) SIM, SATISFAZEM

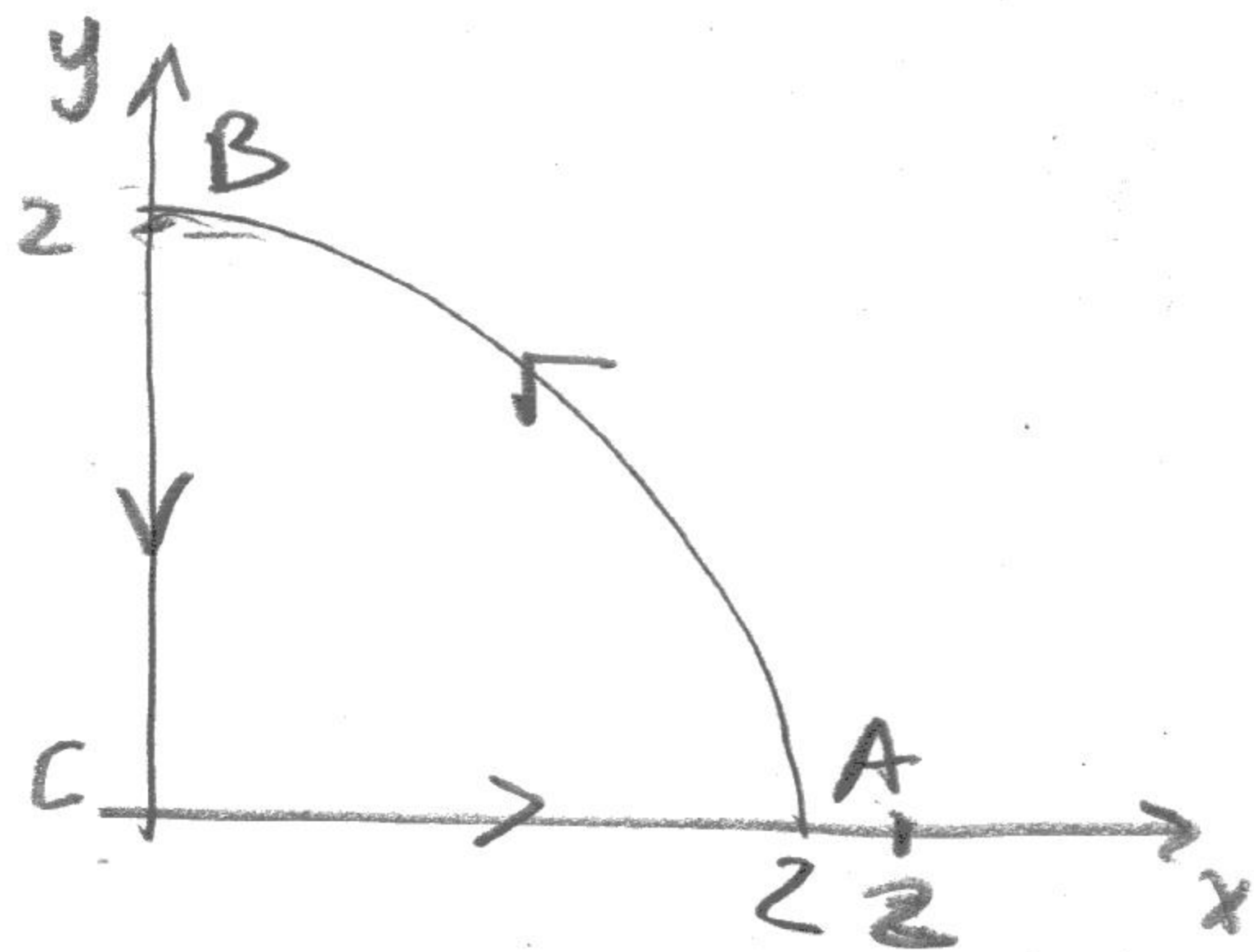
NO ITEM a) O VALOR É POSITIVO PORQUE
A INTEGRAL ESTÁ SENDO FEITA NO SENTIDO HORÁRIO.
ASSIM, DEVE SER COMPARADA COM UM VETOR ~~ÁREA~~
~~PARTE~~ DENTRO DO PAPEL (REGRAS DA MÃO DIREITA).
NO ITEM b) A ÁREA ESTÁ COM O SEU VETOR
NO SENTIDO POSITIVO DO EIXO Z QUE É PARA
FORA DO PAPEL, POR ISSO OS SINAIS SÃO
CONTRÁRIOS.

$$3) \vec{A} = \rho \sin \varphi \hat{a}_\rho + \rho^2 \hat{a}_\varphi$$

$$\vec{A} \rightarrow B \quad d\vec{l} = d\rho \hat{a}_\rho + \rho d\varphi \hat{a}_\varphi \quad \rho = 2$$

$$\vec{A} \cdot d\vec{l} = \rho^2 d\varphi \quad \int_A^B \vec{A} \cdot d\vec{l} = \int_0^{\pi/2} 2^3 d\varphi$$

$$\int_A^B \vec{A} \cdot d\vec{l} = 4\pi$$



$$B \rightarrow C \quad d\vec{l} = d\rho \hat{a}_\rho + \rho d\varphi \hat{a}_\varphi \quad \varphi = \pi/2$$

$$\vec{A} \cdot d\vec{l} = \rho \sin \varphi d\rho \quad \int_B^C \vec{A} \cdot d\vec{l} = \int_2^0 \rho \sin \pi/2 d\rho = -2$$

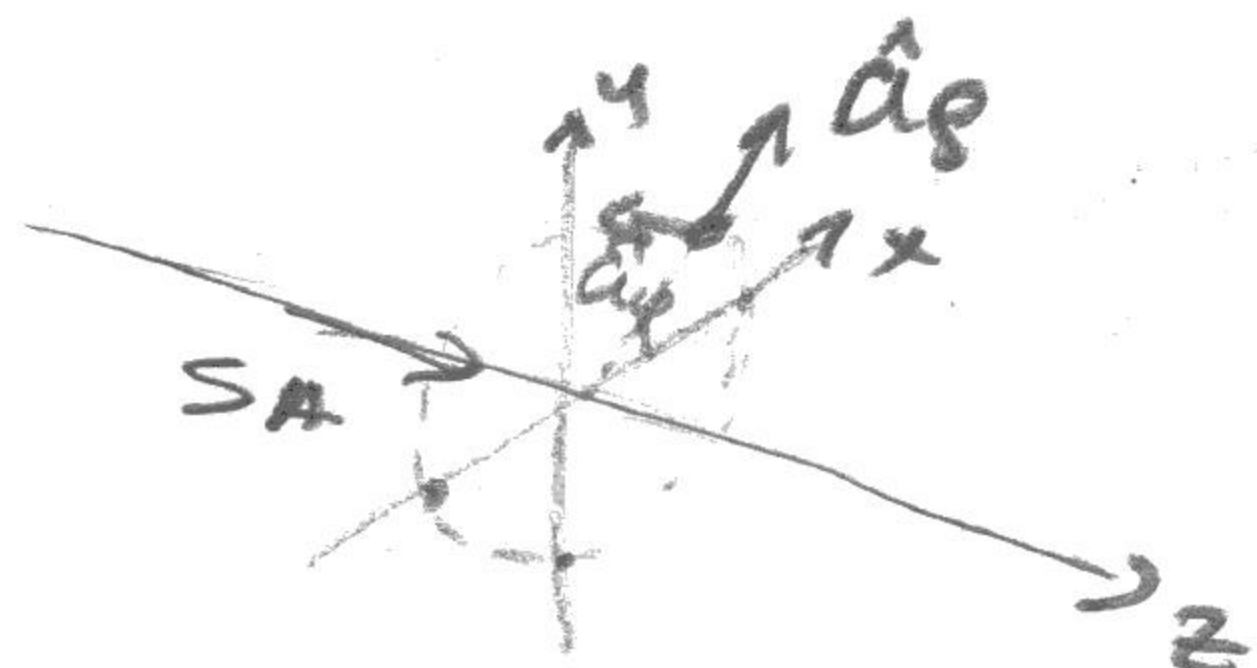
$$C \rightarrow A \quad d\vec{l} = d\rho \hat{a}_\rho + \rho d\varphi \hat{a}_\varphi \quad \varphi = 0$$

$$\int_C^A \vec{A} \cdot d\vec{l} = \int_0^2 \rho \sin 0 d\rho = 0$$

$$\oint \vec{A} \cdot d\vec{l} = -2 + 4\pi$$

4) PARA O FIO FINO INFINITO:

$$\vec{H} = \frac{I}{2\pi r} \hat{a}_\phi$$

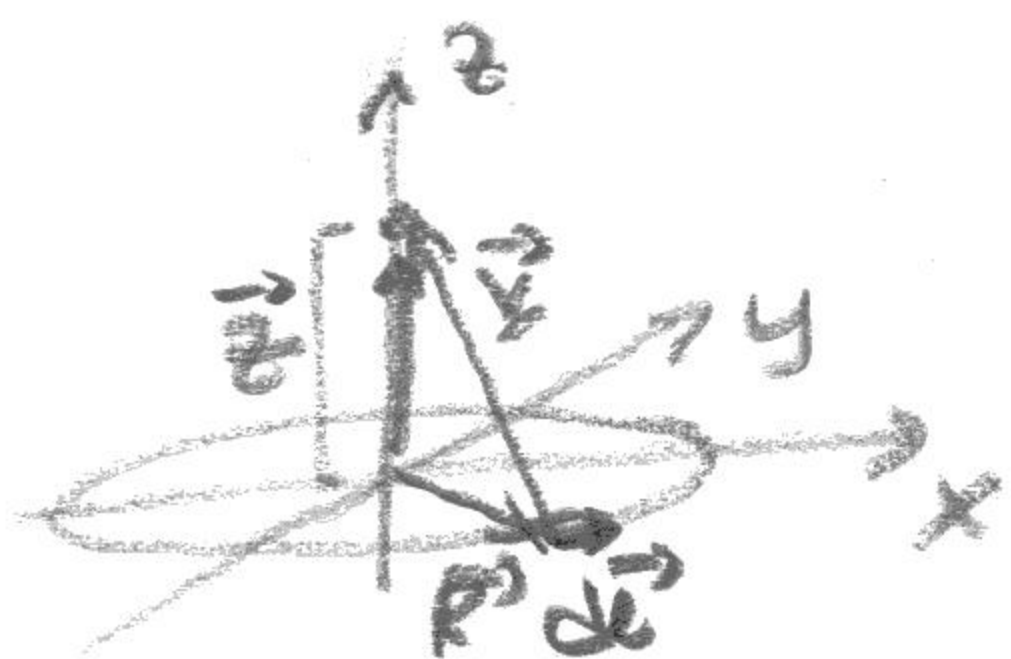


$$(3, 0, 0) \rightarrow r = 3\text{ m} \Rightarrow \vec{H} = \frac{5}{2\pi 3} = 0,27 \hat{a}_\phi \text{ A/m}$$

$$(0, 3, 5) \rightarrow r = 3\text{ m} \Rightarrow \vec{H} = 0,27 \hat{a}_\phi \text{ A/m}$$

$$(1, 1, 2) \rightarrow r = \sqrt{2}\text{ m} \Rightarrow \vec{H} = \frac{5}{2\pi \sqrt{2}} \hat{a}_\phi = 0,57 \hat{a}_\phi \text{ A/m}$$

5) CAMPO MAGNÉTICO PRODUZIDO POR UMA ESPIRA CIRCULAR NUM PONTO NO EIXO DE SIMETRIA



$$d\vec{H} = \frac{I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3} \quad \vec{r} = \vec{z} - \vec{R} = z\hat{a}_z - R\hat{a}_\rho$$

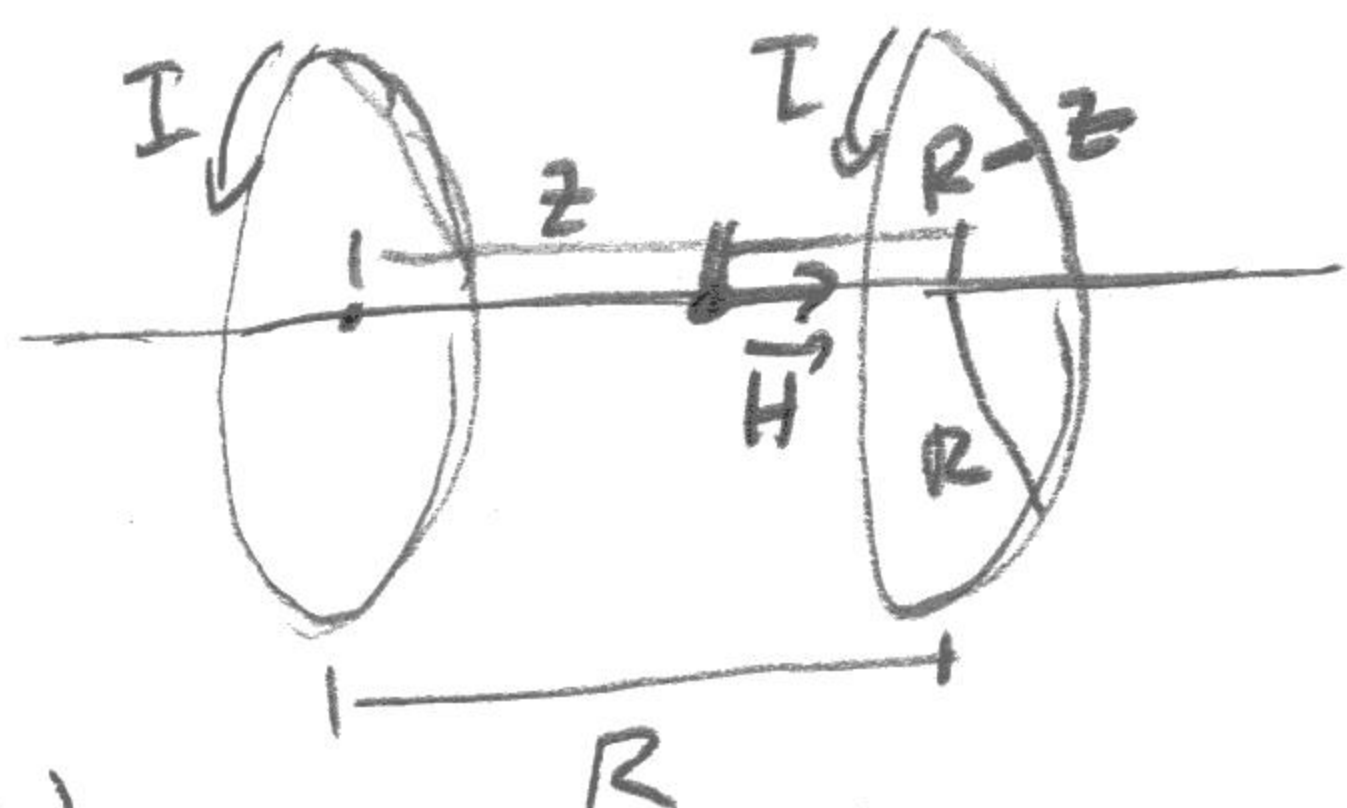
$$r = \sqrt{z^2 + R^2} \quad dl = R d\varphi$$

$$d\vec{l} = dl \hat{a}_\varphi \quad d\vec{l} \times \vec{r} = (z \hat{a}_\rho \times \hat{a}_z - R \hat{a}_\varphi \times \hat{a}_\rho) dl = (z \hat{a}_\varphi + R \hat{a}_z) dl$$

$$d\vec{H} = \frac{I}{4\pi} \frac{(z \hat{a}_\varphi + R \hat{a}_z) dl}{(z^2 + R^2)^{3/2}} \quad \vec{H} = \frac{I}{4\pi} \left[\int_0^{2\pi} \frac{z R \hat{a}_\varphi d\varphi}{(z^2 + R^2)^{3/2}} + \int_0^{2\pi} \frac{R^2 \hat{a}_z d\varphi}{(z^2 + R^2)^{3/2}} \right]$$

$$\vec{H} = \frac{IR^2}{2(z^2 + R^2)^{3/2}} \hat{a}_z$$

AS DUAS ESPIRAS, CUJAS CORRES SÃO GERADAS NO MESMO SENTIDO GERAM CAMPO NO MESMO SENTIDO NO EIXO DE SIMETRIA.



$$\vec{H}_T = \frac{IR^2}{2} \left\{ \frac{1}{(z^2 + R^2)^{3/2}} + \frac{1}{[(R-z)^2 + R^2]^{3/2}} \right\} \hat{a}_z$$

$$b) \frac{dH_T}{dz} = \frac{IR^2}{2} \left\{ -3 \frac{z}{(z^2 + R^2)^{5/2}} + 3 \frac{(R-z)}{[(R-z)^2 + R^2]^{5/2}} \right\}$$

$$\frac{d^2 H_T}{dz^2} = \frac{3IR^2}{2} \left\{ \frac{4z^2 - R^2}{(z^2 + R^2)^{7/2}} + \frac{4(R-z)^2 - R^2}{[(R-z)^2 + R^2]^{7/2}} \right\}$$

ESTA CONFIGURAÇÃO É CHAMADA DE BOBINA DE HELMHOLTZ. NOTE QUE HÁ UMA REGIÃO BEM GRANDE EM TORNO DO PONTO ENTRE AS DUAS BOBINAS QUE O CAMPO É PRATICAMENTE HOMOGÊNEO PORQUE A DERIVADA NO CENTRO É ZERO E A 2ª DERIVADA TAMBÉM.

$$6) \quad d\vec{r} = dx \hat{a}_x + dy \hat{a}_y \quad y = -2x + 2$$

$$\vec{r}' = -x \hat{a}_x - y \hat{a}_y \quad r = \sqrt{x^2 + y^2}$$

$$d\vec{H} = \frac{I}{4\pi} \frac{d\vec{r}' \times \vec{r}'}{r^3} = \frac{I}{4\pi} \frac{(-y dx + x dy) \hat{a}_z}{(x^2 + y^2)^{3/2}}$$

$$dy = -2dx$$

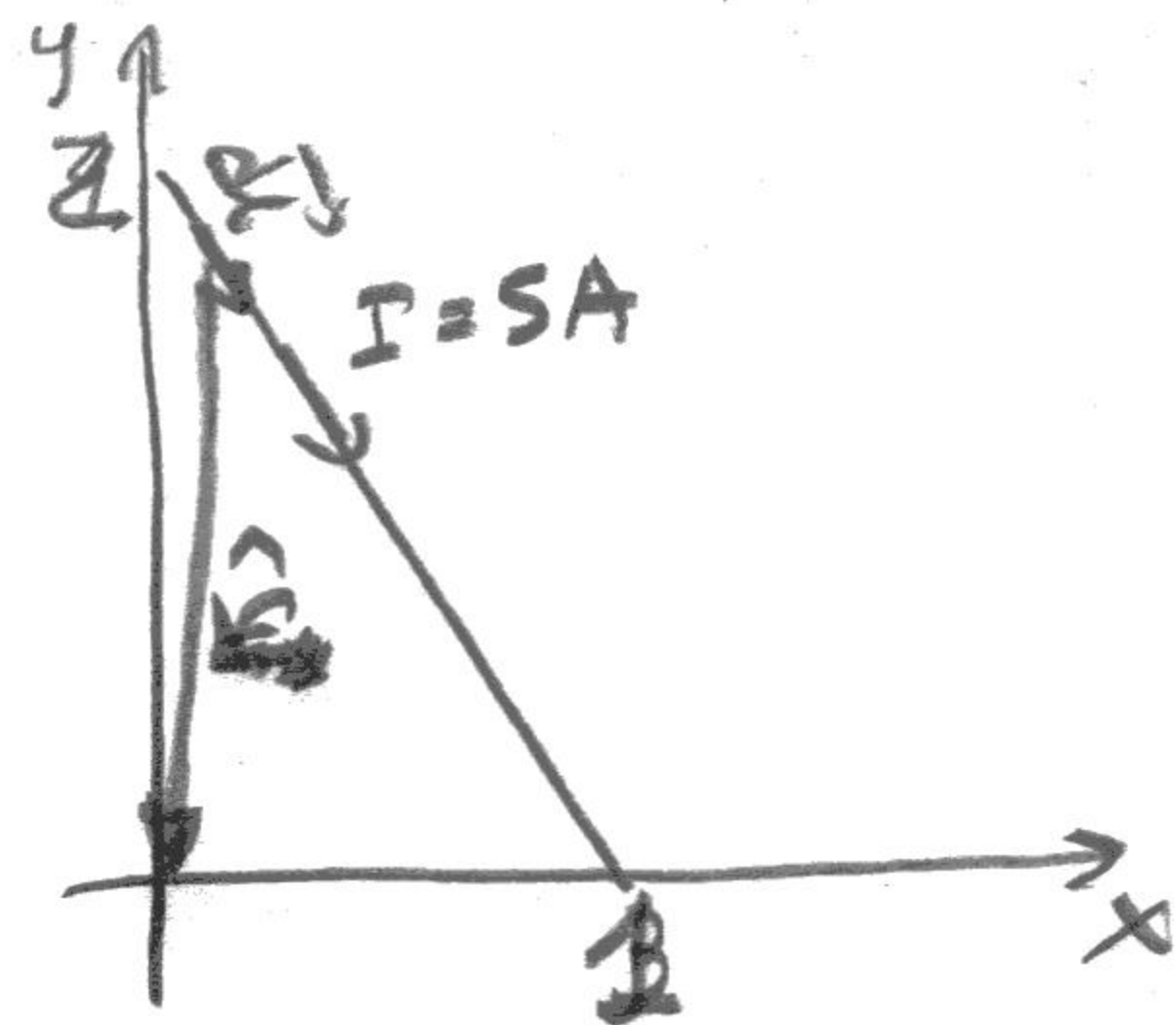
$$dH = \frac{I}{4\pi} \frac{[-(-2x+2)dx - 2x dx]}{[x^2 + (-2x+2)^2]^{3/2}} = \frac{I}{4\pi} \frac{-2}{[5x^2 - 8x + 4]^{3/2}} dx$$

$$\text{SE } x = \frac{\mu + 8}{10} \quad dH = \frac{-I}{2\pi} \frac{1}{(\mu^2 + \frac{4}{5})^{3/2}} \frac{d\mu}{10} \quad \begin{matrix} x=0 & \mu = -8 \\ x=1 & \mu = 2 \end{matrix}$$

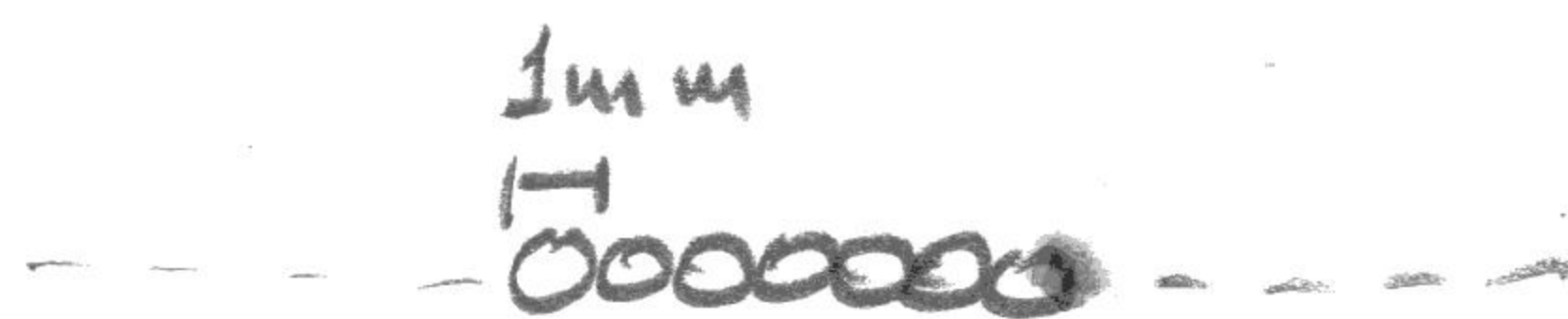
$$H = \frac{-I}{20\pi} \int_{-8}^2 \frac{d\mu}{(\mu^2 + \frac{4}{5})^{3/2}} = -\frac{I}{20\pi} \left. \frac{\mu}{\frac{4}{5} \sqrt{\mu^2 + \frac{4}{5}}} \right|_{-8}^2 =$$

$$H = -\frac{5}{16\pi} \left[\frac{2}{\sqrt{4 + \frac{4}{5}}} + \frac{8}{\sqrt{64 + \frac{4}{5}}} \right] = -0,19 \text{ A/m}$$

$$\vec{H} = -0,19 \hat{a}_z \text{ A/m}$$



7)



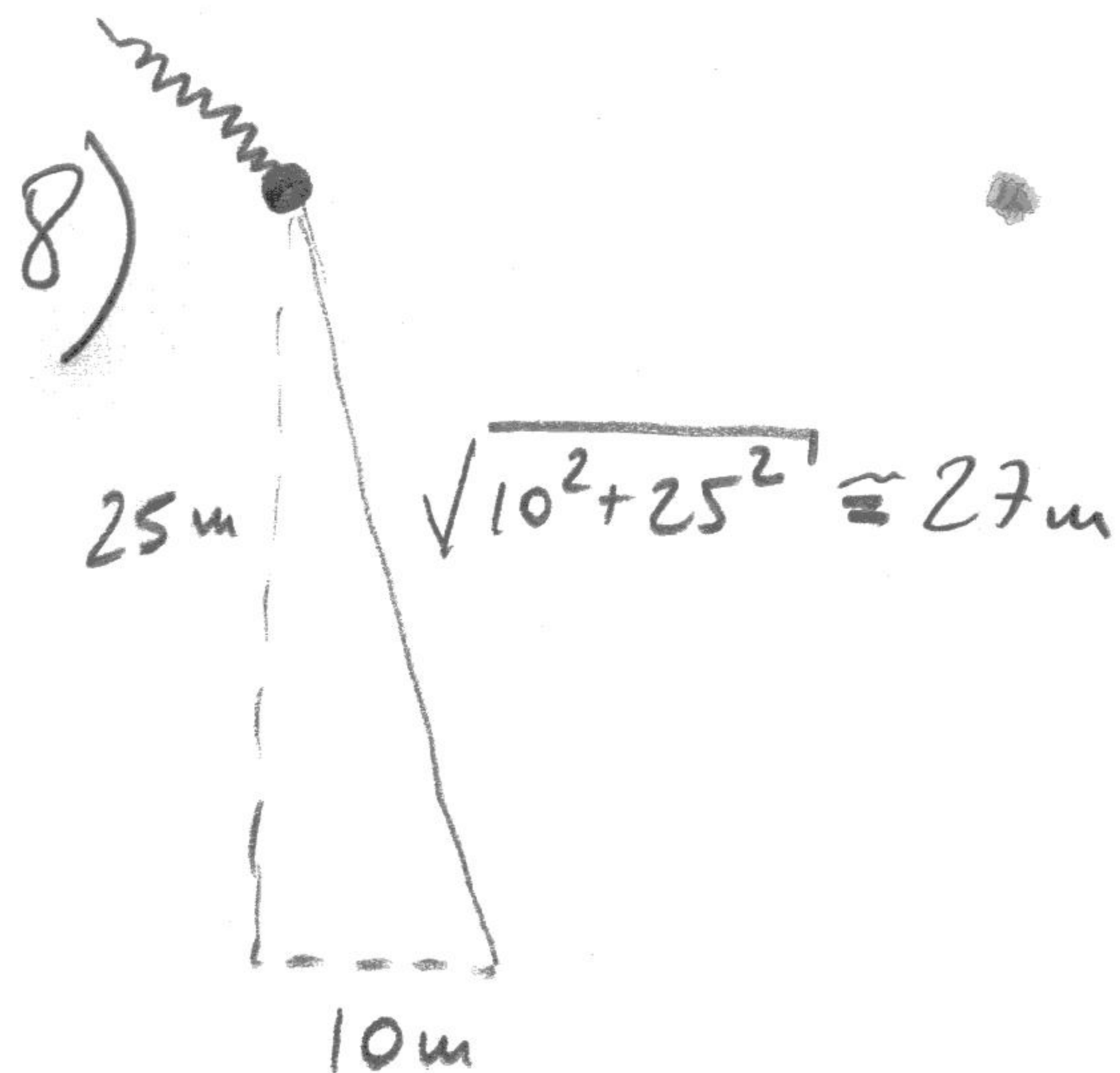
DENSIDADE DE ESPIRAS

$$n = \frac{1}{d} = \frac{1}{1\text{mm}} = 1000 \text{ est/m}$$



$$\theta = \mu_0 n i_0 = 4\pi \times 10^{-7} \times 10^3 \times i_0 = 1$$

$$i_0 = \frac{1}{4\pi} 10^4 = \underline{796 \text{ A}}$$



$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 2 \times 10^3}{2\pi \cdot 27}$$

$$B = 1,48 \times 10^{-5} \text{ T} = 0,148 \text{ G}$$

PARA COMPARAÇÃO, ESTE CAMPO É DA ORDEM DE GRANDEZA DO CAMPO MAGNÉTICO DA TERRA. UMA BÚSSOLA NESTA POSIÇÃO IRÁ DEFLETIR POR CAUSA DO CAMPO DO CABO

9)

a) $I_1 = I_2$ POIS EM ESTADO ESTACIONÁRIO
A QUANTIDADE DE CARGA POR UNIDADE DE TEMPO
QUE ENTRA NÃO PODE SER DIFERENTE QUANTO SAÍ

$$b) j = \frac{I}{A} \quad j_1 = \frac{I}{A_1} \quad j_2 = \frac{I}{A_2} \quad A_1 < A_2$$

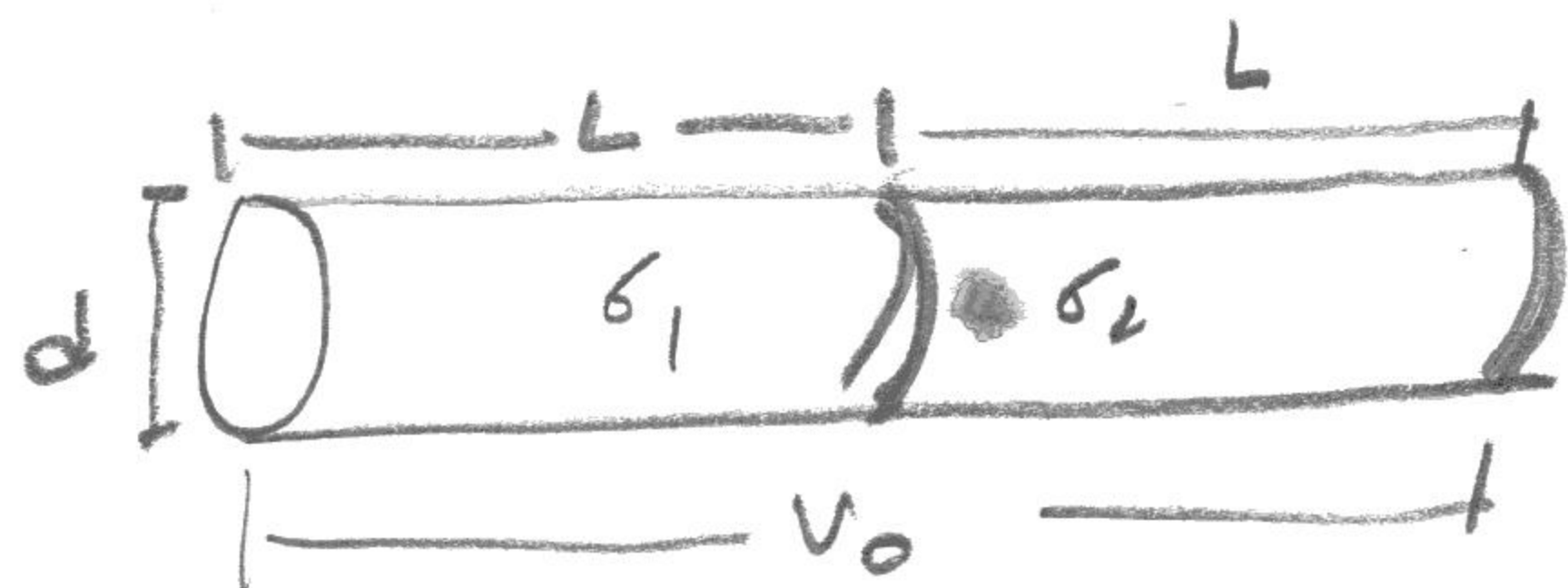
$$\text{ENTÃO } j_1 > j_2$$

c) $j = \sigma E$ como são do mesmo MATERIAL
 $\sigma_1 = \sigma_2$ como $j_1 > j_2$ $E_1 > E_2$

d) $j = nev$ como são do mesmo MATERIAL
 $n_1 = n_2$ como $j_1 > j_2$ $v_1 > v_2$

MAIOR VELOCIDADE PARA PASSAR
A MESMA QUANTIDADE DE CARGA POR UNIDADE
DE TEMPO

10)



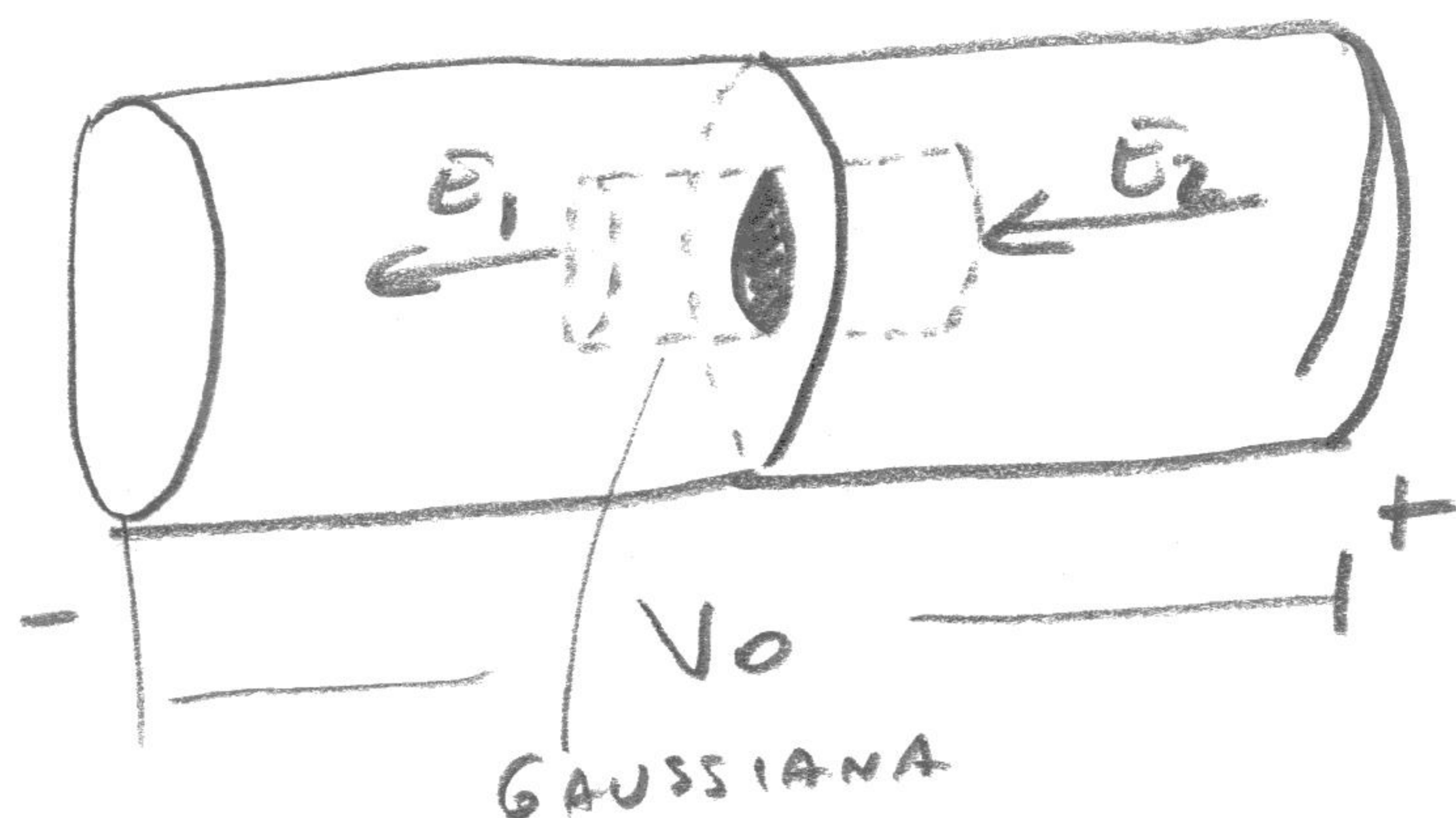
$$R_1 = \rho \frac{L}{A} = \frac{4L}{\epsilon_1 \pi d^2}$$

$$R_2 = \frac{4L}{\epsilon_2 \pi d^2}$$

$$R_T = \frac{4L}{\pi d^2} \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right) \quad I = \frac{V_0}{R_T} = \frac{V_0 \pi d^2}{4L} \left(\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} \right)$$

$$j = \frac{I}{A} = \frac{4I}{\pi d^2} = \frac{V_0}{L} \left(\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} \right) \quad j_1 = j_2$$

$$j = \sigma E \quad E_1 = \frac{j}{\epsilon_1} = \frac{V_0}{L} \left(\frac{\epsilon_2}{\epsilon_1 + \epsilon_2} \right) \quad E_2 = \frac{V_0}{L} \left(\frac{\epsilon_1}{\epsilon_1 + \epsilon_2} \right)$$



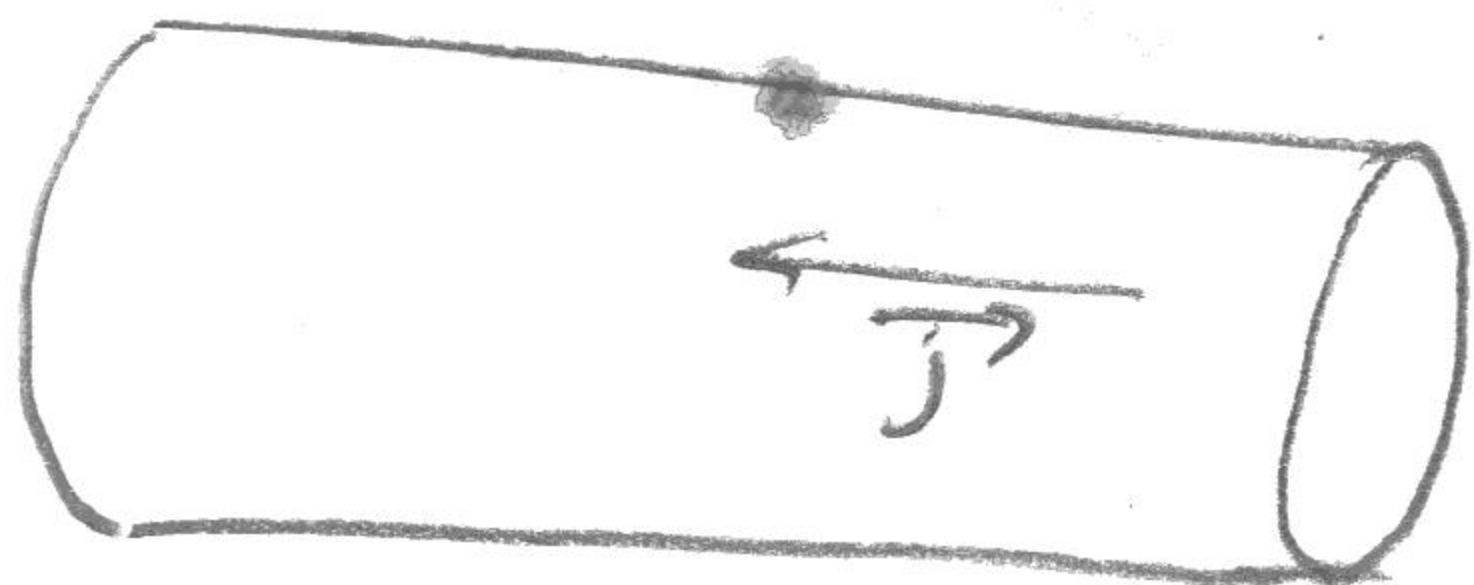
TRAÇANDO UMA GAUSSIANA INCLUINDO O CONTORNO ENTRE OS CONDUTORES

$$\oint \vec{E} \cdot d\vec{S} = E_1 A - E_2 A = Q_S A$$

$$Q_S = E_1 - E_2 = \frac{V_0}{L} \left(\frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} \right)$$

A CARGA DEPOSITADA NO CONTORNO ENTRE OS CONDUTORES ALTERA OS CAMPOS ELÉTRICOS DOS DOIS LADOS.

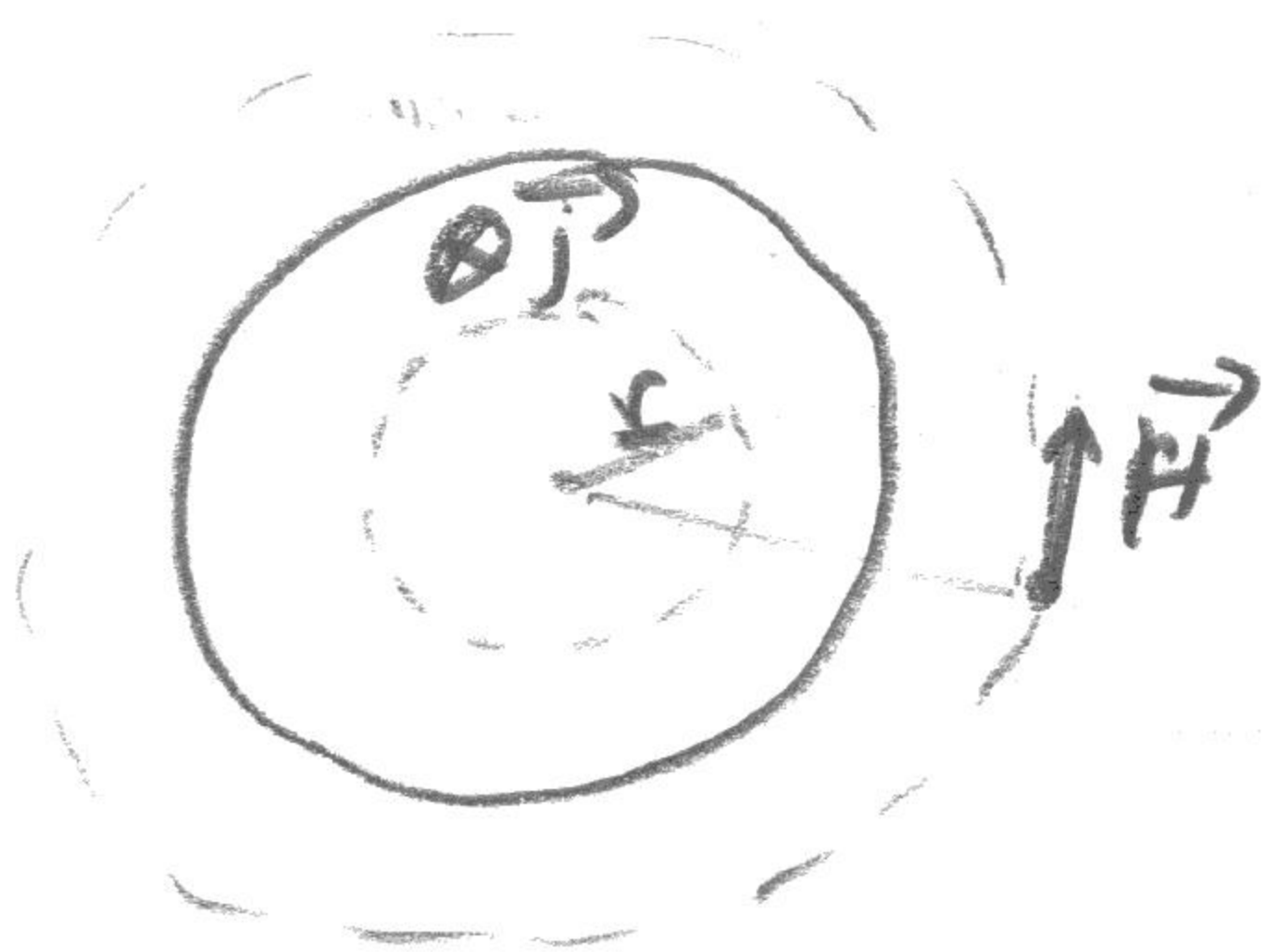
11)



$$a) \quad I = \int j \, dA = \int_0^R \left(J_0 - C \frac{r^2}{R^2} \right) 2\pi r \, dr =$$

$$= 2\pi J_0 \frac{r^2}{2} \Big|_0^R - 2\pi \frac{C}{R^2} \frac{r^4}{4} \Big|_0^R = 2\pi R^2 \left(\frac{J_0}{2} - \frac{C}{4} \right)$$

b)



$$r > R \quad \oint \vec{H} \cdot d\vec{l} = I_{int}$$

$$H 2\pi r = 2\pi R^2 \left(\frac{J_0}{2} - \frac{C}{4} \right)$$

$$\vec{H} = \frac{R^2}{r} \left(\frac{J_0}{2} - \frac{C}{4} \right) \hat{a}_\varphi$$

$$r < R \quad \oint \vec{H} \cdot d\vec{l} = I_{int}$$

$$I_{int} = \int_0^r \left(J_0 - \frac{C r^2}{R^2} \right) 2\pi r \, dr$$

$$I_{int} = 2\pi J_0 \frac{r^2}{2} - 2\pi C \frac{r^4}{4R^2}$$

$$H = J_0 \frac{r}{2} - C \frac{r^3}{4R^2}$$

$$\vec{H} = \frac{r}{2} \left(J_0 - \frac{C r^2}{2R^2} \right) \hat{a}_\varphi$$

$$c) \vec{B} = \mu_0 \vec{H} = \vec{\nabla} \times \vec{A} = \left(\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} \right) \hat{a}_\varphi$$

por simetria

$$\frac{\partial A_\varphi}{\partial z} = 0$$

$$\frac{\partial A_z}{\partial \varphi} = \mu_0 H_\varphi$$

$$r < R \quad \frac{dA_z}{dr} = \mu_0 \left(\frac{J_0 r}{2} - \frac{C r^3}{4 R^2} \right)$$

$$A_z = \mu_0 \left(\frac{J_0 r^2}{4} - \frac{C r^4}{16 R^2} \right) + A_0$$

$$r > R \quad \frac{dA_z}{dr} = \mu_0 \frac{R^2}{r} \left(\frac{J_0}{2} - \frac{C}{4} \right)$$

$$A_z = \mu_0 R^2 \left(\frac{J_0}{2} - \frac{C}{4} \right) \ln r + A_1$$

$$r = R \quad A_{z \text{ fora}} = A_{z \text{ dentro}}$$

$$\mu_0 \left(\frac{J_0 R^2}{4} - \frac{C R^2}{16} \right) + A_0 = \mu_0 \left(\frac{J_0}{2} - \frac{C}{4} \right) \ln R + A_1$$

Podemos fazer $A_z(R) = 0$ assim

$$A_1 = -\mu_0 \left(\frac{J_0}{2} - \frac{C}{4} \right) \ln R$$

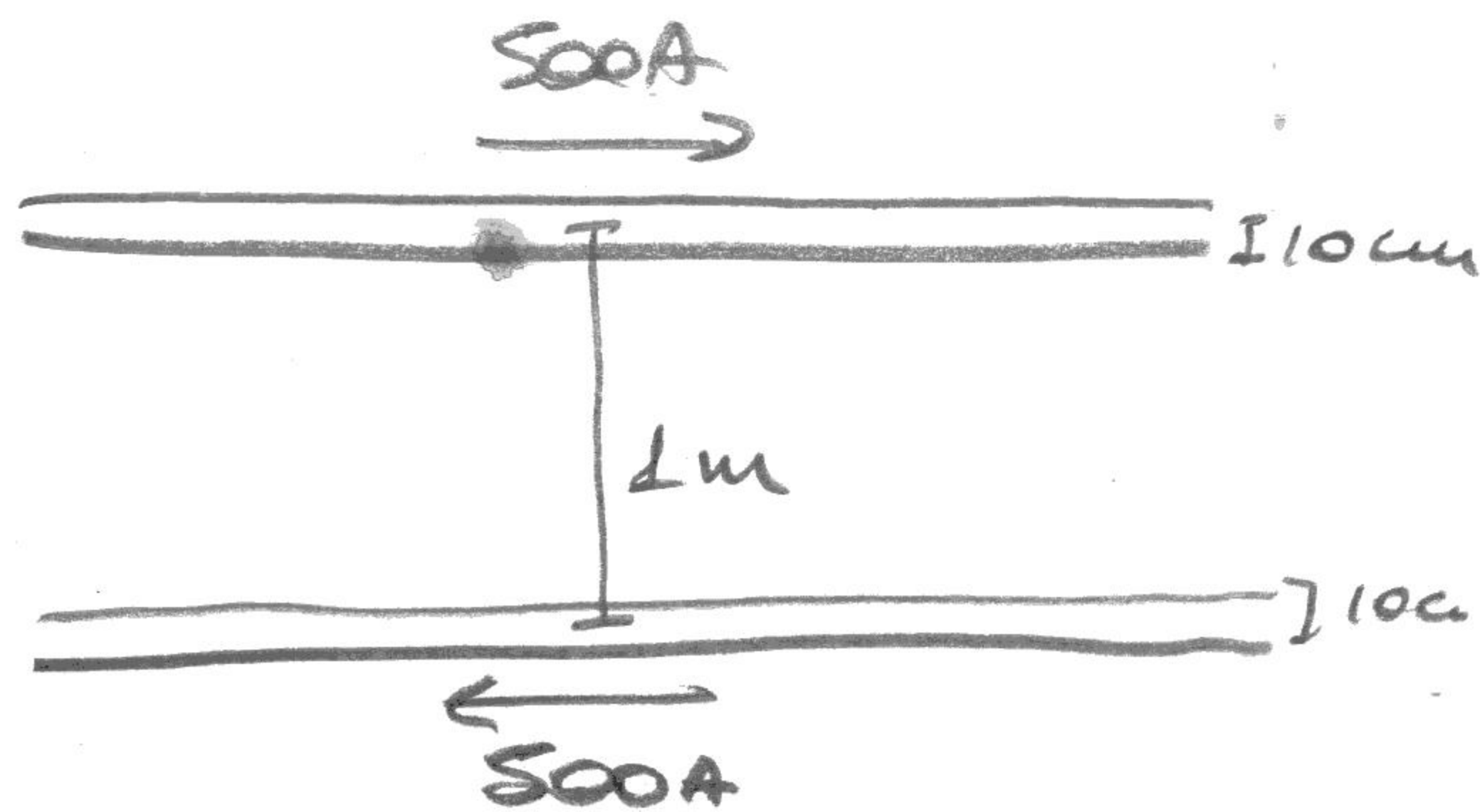
então

$$A_0 = -\mu_0 \left(\frac{J_0 R^2}{4} - \frac{C R^2}{16} \right)$$

d)

$$\vec{\nabla} \times \vec{A} = \frac{\partial A_z}{\partial r} \hat{a}_\varphi = B \hat{a}_\varphi$$

12)



$$B = \frac{\mu_0 I}{2\pi r}$$

$$H = \frac{I}{2\pi r}$$

$$B = \frac{4\pi \times 10^{-7} \times 5 \times 10^2}{2\pi \times 1} = 1 \times 10^{-4} \text{ T}$$

$$F = I L B \Rightarrow \frac{F}{L} = I B = 500 \times 1 \times 10^{-4} = 0,05 \frac{\text{N}}{\text{m}}$$